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ASSESSMENT OF RELIABILITY FOR REPAIRABLE SYSTEMS.(U)

**F/G 12/1**

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ASSESSMENT OF RELIABILITY FOR REPAIRABLE SYSTEMS, (U)  
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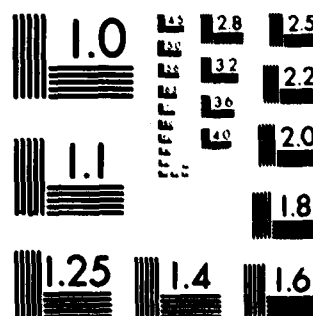
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| <p>Define <math>Z(t)</math> to be the forward recurrence time at <math>t</math> for a renewal process with interarrival time distribution, <math>F</math>, which is assumed to be IMRL (increasing mean residual life). It is shown that <math>E[Z(t)]</math> is increasing in <math>t \geq 0</math> for all increasing convex <math>F</math>. An example demonstrates that <math>Z(t)</math> is not necessarily stochastically increasing nor is the renewal function necessarily concave. Both of these properties are known to hold for <math>F</math> DFR (decreasing failure rate).</p> |  |

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**AFOSR-TR- 80 - 1341**

**Assessment of Reliability for Repairable Systems**

**Principal Investigator: Mark Brown**

**Statistics Report M-563**

**AFOSR Technical Report 79 B-7**

**November, 1980  
Department of Statistics  
The Florida State University**

**Final Technical Report on [REDACTED] F49620-79-C-0157.**

**Grant Period: July 1, 1979 through September 30, 1980.**

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PREFACE

This is the final technical report for AFOSR grant F49620-79-C-0157 "Assessment of Reliability for Repairable Systems." The grant period was July 1, 1979 through September 30, 1980.

In this final report we:

a) List specifically the accomplishments in terms of technical reports that have been written, and articles that have been published or are scheduled to appear.

b) Give for each paper and/or technical report, a technical summary (abstract), and a non-technical summary with the latter describing the significance of the research.

c) List the specific activities and honors of note of the principal investigator during the contract period.

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### 1. List of Accomplishments

Technical reports produced under grant F49620-79-C-0157.

- 1 - Further monotonicity properties for specialized renewal processes, by Mark Brown, July 1979.
- 2 - Approximating DFR distributions by exponential distributions, with applications to first passage times, by Mark Brown, August 1979.
- 3 - On the choice of variance for the log rank test, by Mark Brown, November 1979.
- 4 - Randomly evolving hazard rate functions, by Mark Brown and Sheldon M. Ross, March 1980.
- 5 - Imperfect repair, by Mark Brown and Frank Proschan, April 1980.
- 6 - On the first passage time distribution for a class of Markov chains, by Mark Brown and C.N. Rao, July 1980.

### Publication of above reports

Reports (1) and (2) above have been accepted for publication by the Annals of Probability. Report (3) has been accepted for publication by Biometrika. Reports (4), (5) and (6) have been submitted for publication.

## 2 - Discussion of papers

### 2.1) Further monotonicity properties for specialized renewal processes.

#### 2.1.a) Abstract

Define  $Z(t)$  to be the forward recurrence time at  $t$  for a renewal process with interarrival time distribution,  $F$ , which is assumed to be IMRL (increasing mean residual life). It is shown that  $E\phi(Z(t))$  is increasing in  $t \geq 0$  for all increasing convex  $\phi$ . An example demonstrates that  $Z(t)$  is not necessarily stochastically increasing nor is the renewal function necessarily concave. Both of these properties are known to hold for  $F$  DFR (decreasing failure rate).

#### 2.1.b) Significance of the research

Renewal theory has been widely applied to maintenance and replacement of equipment. A renewal is interpreted as a replacement of a failed component by a new component, and the interarrival time distribution is the time until failure of a new component.

Traditional renewal theory derives asymptotic properties of renewal processes under moment conditions and smoothness requirements for the interarrival time distribution. If further information about the distribution is known (for example monotonicity and aging properties) there is in general no way to exploit this information to obtain sharper and more refined results. Thus the need arises for obtaining strengthened renewal results under stronger assumptions on the interarrival time distribution. Furthermore, for practical purposes, bounds and inequalities are needed for fixed  $t$ , and these do not follow from the asymptotic theory.

In this paper and in the paper "Bounds, inequalities and monotonicity properties for specialized renewal processes", Annals of Probability, (1980),

8, 227-40, the principal investigator achieves the above objectives for the cases of IMRL (increasing mean residual life) and DFR (decreasing failure rate) distributions.

### 2.1.c. Status of paper

The paper has been accepted for publication by the Annals of Probability.

2.2) Approximating DFR distributions by exponential distributions, with applications to first passage times.

### 2.2.a) Abstract

It is shown that if  $F$  is a DFR (decreasing failure rate) distribution on  $[0, \infty)$  then:

$$\max_t (\sup_t |\bar{F}(t) - \bar{G}(t)|, \sup_t |\bar{F}(t) - e^{-t/\mu}|, \sup_t |\bar{G}(t) - e^{-t/\mu}|) \leq 1 - e^{-\rho}$$

where  $\bar{F}(t) = 1 - F(t)$ ,  $\mu = E_F X$ ,  $\mu_2 = E_F X^2$ ,  $\bar{G}(t) = \frac{1}{\mu} \int_t^\infty \bar{F}(x) dx$ ,  $\mu_G = E_G X$ ,

and  $\rho = \frac{\mu_2}{2\mu^2} - 1 = \frac{\mu_G}{\mu} - 1$ . Thus if  $F$  is DFR and  $\rho$  is small then  $F$  and  $G$  are approximately equal and exponential with parameter  $1/\mu$ . DFR distributions with small  $\rho$  arise naturally in a class of first passage time distributions for Markov processes, as first illuminated by Keilson. The current results thus provide error bounds for exponential approximation of these distributions.

### 2.2.b) Significance of the Research

||

In the quantitative analysis of reliability for systems with repairable components, the goal is to approximate the distribution of the time to first system failure, given the failure and repair rates of the components and the fault tree structure of the system.

Keilson, in a series of papers, has shown that two important first passage times arising in the study of repairable systems have DFR distributions. Moreover, he argues that these distributions should be approximately



exponential. From his work the following problem arises: Given a DFR distribution with first two moments known, obtain a uniform bound between this distribution and an exponential distribution with the same mean.

The current paper provides a bound which substantially improves upon previous bounds obtained by Keilson and Steutel, Heyde, Heyde and Leslie, and Hall. Moreover the bound applies to a larger class of distributions and is very close to the best possible bound.

#### 2.2.c) Status of paper

The paper has been accepted for publication by the Annals of Probability

#### 2.3) On the choice of variance for the log rank test.

##### 2.3.a) Abstract

The log rank test is widely used for comparison of survival curves. This paper examines various estimators for the variance of the log rank test statistic. These include the Mantel-Haenszel variance, the permutation variance of Peto and Peto and several newly proposed estimators. The results generalize to a wide class of test statistics for the two sample problem with censored data.

##### 2.3.b) Significance of the Research

The log rank test is currently used at medical centers throughout the country. It provides a non-parametric test of the hypothesis that two samples have the same distribution, for data which is subject to right censoring. A typical application has a control group and experimental group of cancer patients. Chemotherapies are administered with the goal of inducing a lengthy remission period. The control group receives a standard therapy and the experimental group a new therapy. Each patient's observation is his length of time in remission. If a patient relapses after 3 months his observation is  $X = 3$ ; if he received treatment 3 months

ago and has remained in remission until the current time his observation is  $X = 3+$ . The latter observation is known as right censored. We know that  $X > 3$ , but do not know the value of  $X$ . In general the data consists of both uncensored and right censored observations, and the times of entry to the study (and therefore the censoring times) vary from patient to patient, and may be quite different between the two groups.

The current paper provides an improvement in the estimation of the variance of the log rank test statistic. This should lead to more accurate approximation of P values, than that obtained using the Mantel-Haenszel variance.

The techniques for handling censored data currently used in biomedical applications are also applicable to reliability. Instead of studying remission times of patients we can study failure times of equipment. The connection between the fields of Reliability and Biometry was the topic of a 1973 symposium at Florida State University organized by Frank Proschan and Robert Serfling.

### 2.3.c) Status of paper

The paper has been accepted for publication by Biometrika.

### 2.4) Randomly evolving hazard rate functions

#### 2.4.a) Abstract

Abstract Let  $X$  be a positive random variable with cdf  $F$ , and pdf  $f$ . The hazard rate function of  $X$  evaluated at  $t$  is defined by  $h(t) = \frac{f(t)}{1 - F(t)}$ .

It is well known and easy to prove that for  $F$  continuous,  $H(X) = \int_0^X h(t)dt$ , is exponentially distributed with mean 1. Thus the amount of hazard

overcome in a lifetime has the same distribution independent of  $h$ , the rate at which hazard is encountered.

In many models there naturally occurs a random function,  $r(t)$ , which measures the intensity of an event of specified type occurring at time  $t$  given information about the behavior of the system in  $[0, t)$ . We call such a function a randomly evolving hazard rate function. We conjecture that in great generality  $\int_0^X r(t)dt$  is exponential with mean 1.

Several examples are given for which the conjecture holds. A heuristic proof is given for the general case.

#### 2.4.b) Significance of the research

An interesting new principle of aging is suggested (but not quite proved in full generality). The amount of cumulative hazard experienced until death or failure is exponentially distributed with parameter 1 independent of the random rate at which hazard appears. A special case of this result due to Barlow and Proschan is the key idea underlying the total time on test methodology, a powerful tool for goodness of fit for censored data.

2.4.c) Status of paper. The paper has been submitted for publication to the Naval Research Logistics Quarterly.

#### 2.5) Imperfect Repair

##### 2.5.a) Abstract

A device is repaired at failure. With probability  $p$ , it is returned to the "good as new" state (perfect repair), with probability  $1 - p$  it is returned to the functioning state, but it is only as good as a device of age equal to its age at failure (imperfect repair). Repair takes negligible time. We obtain the distribution  $F_p$  of the interval between successive good as new states in terms of the underlying life distribution  $F$ . We

show that if  $F$  is in any of the life distribution classes: IFR, DFR, IFRA, DFRA, NBU, NWU, DMRL, or IMRL, then  $F_p$  is in the same class. Finally, we obtain a number of monotonicity properties for various parameters and random variables of the stochastic process. The results obtained are of interest in the context of stochastic processes in general, as well as being useful in the particular imperfect repair model studied.

#### 2.5.b) Significance of the Research

The underlying assumption in renewal theory approaches to replacement and repair is that a failed component is replaced by a new component. In practice a failed component may be repaired, resulting in a working component which is not as good as new. Models are needed which incorporate this imperfect repair. This paper allows both for perfect repairs (usually resulting from replacing a failed component by a new component) and imperfect repairs. Research continues by the authors on still more realistic models.

#### 2.5.c) Status of paper

The paper has been submitted for publication to the Journal of Applied Probability.

### 2.6) On the first passage time distribution for a class of Markov chains.

#### 2.6.a) Abstract

Consider a stochastically monotone Markov chain with monotone paths on a partially ordered countable set  $S$ . Let  $C$  be an increasing subset of  $S$  with finite complement. Then the first passage time from  $i \in S$  to  $C$  is shown to be IFRA (increasing failure rate on the average). Several applications are presented including coherent systems, shock models, and convolutions of IFRA distributions.

### 2.6.b) Significance of the Research

IFRA distributions play an important yet mysterious role in Reliability Theory. The IFRA closure theorem of Birnbaum, Esary and Marshall, the shock model results of Esary, Marshall and Proschan, the IFRA convolution theorem of Block and Savits, and the generalized IFRA closure theorem of Ross, are very significant results which illustrate the remarkable role of IFRA distributions in the study of wear-out and failure.

The current paper attempts to throw light on the question of why IFRA, i.e. what intrinsic property of IFRA distributions is responsible for the above and other results. Our point of view is that IFRA distributions arise as first passage times to increasing sets in stochastically monotone Markov chains on partially ordered sets, and that numerous applications can be interpreted in this context.

### 2.6.c) Status of paper

The paper has been submitted for publication to the Annals of Probability.

3) Activities and honors of the principal investigator during the contract period.

In August 1980 Mark Brown received the honor of becoming an elected Fellow of the Institute for Mathematical Statistics.

Brown's travel activities during the contract period included a December 1979 visit to New York City to discuss Markov process applications to reliability with Professor Cyrus Derman of Columbia University, a visit to Los Angeles, California in February 1980 where he served as a member of a NIH committee to review grant proposals in biostatistics (travel paid by NIH), and a visit to Ottawa, Canada in May 1980 where he presented his paper "The choice of variance for the log rank test" to the International Symposium in Statistics.